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Generalized Maxwell's Equations and Quantum Mechanics. II. Generalized Dirac Equation

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The bare field approximation, in which the results of a recent paper were deduced, is removed. A generalized Dirac equation is obtained. The bare field, in the absence of sources, is identified with the field associated with the neutrino.

1. INTRODUCTION

In a recent paper (Campolattaro, 1989) some preliminary results (Campolattaro, 1989*a*,*b*) have been generalized by a feedback process in the absence of free sources. In what has been called (Campolattaro, 1989) the "bare field approximation," and in the absence of the magnetic monopole, the Dirac equation for the free electron has been deduced.

In the present paper the bare field approximation is removed, both electronic and monopolar currents are considered, and a generalized Dirac equation is deduced. The bare field is identified with the field associated with the neutrino.

2. THE FEEDBACK EFFECT

As was shown in the above-mentioned papers, given an electric current j^{μ} and a monopolar current g^{μ} generating an electromagnetic field $F^{\mu\nu}$ through the generalized Maxwell equations

$$F^{\mu\nu}_{,\nu}=j^{\mu} \tag{1}$$

$$*F^{\mu\nu}_{,\nu}=g^{\mu} \tag{2}$$

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the upper left * in $^*F^{\mu\nu}$ denoting, as usual, the dual tensor, it is possible to find a spinor Ψ such that equations (1) and (2) can be written as follows:

$$(\bar{\Psi}S^{\mu\nu}\Psi)_{,\nu}=j^{\mu} \tag{3}$$

$$(\bar{\Psi}\gamma^5 S^{\mu\nu}\Psi)_{,\nu}=g^{\mu} \tag{4}$$

with the spinor Ψ subjected to the two gauge conditions

$$I_m[\bar{\Psi}\Box\Psi]=0 \tag{5}$$

$$I_m[\bar{\Psi}\gamma^5\Box\Psi]=0 \tag{6}$$

where

$$S^{\mu\nu}=\frac{i}{2}\gamma^{[\mu}\gamma^{\nu]} \tag{7}$$

$$\gamma^{[\mu}\gamma^{\nu]}=\tfrac{1}{2}(\gamma^{\mu}\gamma^{\nu}-\gamma^{\nu}\gamma^{\mu}) \tag{8}$$

$$\gamma^5=\gamma^0\gamma^1\gamma^2\gamma^3 \tag{9}$$

The γ's are the Dirac matrices and $\Box$ is the ordinary d'Alembertian operator.

Equations (3) and (4) have been shown to be completely equivalent to the single nonlinear equation for the spinor Ψ,

$$\gamma^{\mu}\Psi_{,\mu}=-i\gamma^{\mu}\frac{e^{\gamma^5\alpha}}{\rho}\{I_m(\bar{\Psi}_{,\mu}\Psi)-j_{\mu}-\gamma^5[I_m(\bar{\Psi}_{,\mu}\gamma^5\Psi)-g_{\mu}]\}\Psi \tag{10}$$

or, equivalently,

$$\gamma^{\mu}\Psi_{,\mu}=i\gamma^{\mu}\frac{e^{\gamma^5\alpha}}{\rho}\{I_m(\bar{\Psi}\Psi_{,\mu})+j_{\mu}-\gamma^5[I_m(\bar{\Psi}\gamma^5\Psi_{,\mu})+g_{\mu}]\}\Psi \tag{11}$$

with ρ the positive square root of

$$\rho^2=(\bar{\Psi}\Psi)^2+(\bar{\Psi}\gamma^5\Psi)^2 \tag{12}$$

$$e^{\gamma^5\alpha}=\cos\alpha+\gamma^5\sin\alpha \tag{13}$$

and

$$\cos\alpha=\frac{\bar{\Psi}\Psi}{\rho} \tag{14}$$

$$\sin\alpha=\frac{\bar{\Psi}\gamma^5\Psi}{\rho} \tag{15}$$

As described in Campolattaro (1989), the feedback of the two real currents

$$m_{\mu}=m(\bar{\Psi}\gamma_{\mu}\Psi) \tag{16}$$

and

$$n_\mu = -in(\bar{\Psi}\gamma^5\gamma_\mu\Psi) \tag{17}$$

the first one a vector and the latter a pseudovector, with m and n two real scalars such that the conservation conditions

$$m^\mu_{,\mu} = 0 \tag{18}$$

$$n^\mu_{,\mu} = 0 \tag{19}$$

are satisfied, equation (11) is generalized in

$$\gamma^\mu\Psi_{,\mu} = i\gamma^\mu \frac{e^{\gamma^5\alpha}}{\rho}\{I_m(\bar{\Psi}\Psi_{,\mu}) + j_\mu - \gamma^5[I_m(\bar{\Psi}\gamma^5\Psi_{,\mu}) + g_\mu]\}\Psi + i(m - in)\Psi \tag{20}$$

which is equivalent to the generalized Maxwell's equations

$$(\bar{\Psi}S^{\mu\nu}\Psi)_{,\nu} = m^\mu + j^\mu \tag{21}$$

and

$$(\bar{\Psi}\gamma^5 S^{\mu\nu}\Psi)_{,\nu} = n^\mu + g^\mu \tag{22}$$

The bare field approximation consists in studying, instead of equation (20), the equation

$$\gamma^\mu\Psi_{,\mu} = i\gamma^\mu \frac{e^{\gamma^5\alpha}}{\rho}\{j_\mu - \gamma^5 g_\mu\}\Psi + i(m - in)\Psi \tag{23}$$

which is equivalent to studying not the system (21), (22), but the system

$$(\bar{\Psi}S^{\mu\nu}\Psi)_{,\nu} = \eta^{\mu\nu}I_m(\bar{\Psi}\Psi_{,\nu}) + m^\mu + j^\mu \tag{24}$$

and

$$(\bar{\Psi}\gamma^5 S^{\mu\nu}\Psi)_{,\nu} = \eta^{\mu\nu}I_m(\bar{\Psi}\gamma^5\Psi_{,\nu}) + n^\mu + g^\mu \tag{25}$$

with $\eta^{\mu\nu}$ the Minkowski metric tensor, given by

$$\eta^{\mu\nu} = \begin{array}{c} \\ {\scriptstyle 0} \\ {\scriptstyle 1} \\ {\scriptstyle 2} \\ {\scriptstyle 3} \end{array}\!\!\begin{array}{c} \begin{array}{cccc} {\scriptstyle 0} & \;\;{\scriptstyle 1} & \;\;{\scriptstyle 2} & \;\;{\scriptstyle 3} \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \end{array} \tag{26}$$

3. THE MASS FORMULAS

In Campolattaro (1989) it was shown that the masses m and n are given by

$$m = \frac{i}{2}\,\frac{2iA_\mu(\bar{\Psi}\gamma^\mu\Psi) - (\bar{\Psi}\gamma^5\gamma^\mu\Psi)_{,\mu}}{\bar{\Psi}\gamma^5\Psi} \tag{27}$$

and

$$n = \frac{1}{2} \frac{2iA_\mu(\bar{\Psi}\gamma^5\gamma^\mu\Psi) + (\bar{\Psi}\gamma^\mu\Psi)_{,\mu}}{\bar{\Psi}\Psi} \tag{28}$$

with A_μ given by

$$A_\mu = \frac{1}{\rho}(j_\mu \sin\alpha - g_\mu \cos\alpha) \tag{29}$$

Formulas (27) and (28) with (29) were deduced in the bare field approximation. In the general case these mass formulas are still valid provided we make the substitutions

$$j_\mu \to j_\mu + I_m(\bar{\Psi}\Psi_{,\mu}) \tag{30}$$

$$g_\mu \to g_\mu + I_m(\bar{\Psi}\gamma^5\Psi_{,\mu}) \tag{31}$$

so that the mass formulas for the general case are formulas (27) and (28) with the vector A_μ given by

$$A_\mu = \frac{1}{\rho}\{[j_\mu + I_m(\bar{\Psi}\Psi_{,\mu})]\sin\alpha - [g_\mu + I_m(\bar{\Psi}\gamma^5\Psi_{,\mu})]\cos\alpha\} \tag{32}$$

4. VACUUM POLARIZATIONS

In Campolattaro (1989) it was shown that under the gauge conditions (5) and (6), the two currents $I_m(\bar{\Psi}\Psi_{,\mu})$ and $I_m(\bar{\Psi}\gamma^5\Psi_{,\mu})$ define a tensor $P^{\mu\nu}$ through a spinor Φ satisfying

$$P^{\mu\nu} = \bar{\Phi}S^{\mu\nu}\Phi \tag{33}$$

$$*P^{\mu\nu} = \bar{\Phi}\gamma^5 S^{\mu\nu}\Phi \tag{34}$$

$$(\bar{\Phi}S^{\mu\nu}\Phi)_{,\nu} = \eta^{\mu\nu} I_m(\bar{\Psi}\Psi_{,\nu}) \tag{35}$$

and

$$(\Phi\gamma^5 S^{\mu\nu}\Phi)_{,\nu} = \eta^{\mu\nu} I_m(\Psi\gamma^5\Psi_{,\nu}) \tag{36}$$

so that the Maxwell field tensor $F^{\mu\nu}$ is given by

$$F^{\mu\nu} = f^{\mu\nu} + P^{\mu\nu} \tag{37}$$

where $f^{\mu\nu}$ is called the bare field and $P^{\mu\nu}$ the vacuum polarization tensor.

Similarly, by using previous results (Campolattaro, 1989), given the two conserved currents m_μ and n_μ given by equations (16) and (17) with m and n given by equations (27) and (28) with equation (32), one can find a spinor χ and thence a tensor $\Pi^{\mu\nu}$ defined by

$$\Pi^{\mu\nu} = \bar{\chi}S^{\mu\nu}\chi \tag{38}$$

$$*\Pi^{\mu\nu} = \bar{\chi}\gamma^5 S^{\mu\nu}\chi \tag{39}$$

satisfying the equations

$$\Pi^{\mu\nu}_{,\nu} = m^{\mu} = m\bar{\Psi}\gamma^{\mu}\Psi \tag{40}$$

$$*\Pi^{\mu\nu}_{,\nu} = n^{\mu} = -in\bar{\Psi}\gamma^{5}\gamma^{\mu}\Psi \tag{41}$$

so that equations (21) and (22) read

$$(\bar{\Psi}S^{\mu\nu}\Psi - \Pi^{\mu\nu})_{,\nu} = j^{\mu} \tag{42}$$

$$(\bar{\Psi}\gamma^{5}S^{\mu\nu}\Psi - *\Pi^{\mu\nu})_{,\nu} = g^{\mu} \tag{43}$$

In other words, the spinor Ψ solution of equation (20) defines a field $\mathsf{F}^{\mu\nu}$ given by

$$\mathsf{F}^{\mu\nu} = \bar{\Psi}S^{\mu\nu}\Psi \tag{44}$$

such that from equations (42) and (43)

$$\mathsf{F}^{\mu\nu} = F^{\mu\nu} + \Pi^{\mu\nu} \tag{45}$$

$\mathsf{F}^{\mu\nu}$ is the generalized Maxwell electromagnetic field, i.e., the electromagnetic field in the presence of monopolar currents.

The polarization tensor $P^{\mu\nu}$ added to the bare field $f^{\mu\nu}$ gives the ordinary Maxwell field, macroscopic in its nature, called the "macroscopic polarization tensor" (Mpt), while the tensor $\Pi^{\mu\nu}$ will be called the microscopic polarization tensor (mpt).

Moreover, each of these tensors can be decomposed into two tensors in the following way:

$$P^{\mu\nu} = P^{\mu\nu}_{e} + P^{\mu\nu}_{m} \tag{46}$$

$$\Pi^{\mu\nu} = \Pi^{\mu\nu}_{e} + \Pi^{\mu\nu}_{m} \tag{47}$$

satisfying the following equations:

$$(P^{\mu\nu}_{e})_{,\nu} = (\bar{\Phi}_{e}S^{\mu\nu}\Phi_{e})_{,\nu} = \eta^{\mu\nu}I_{m}(\bar{\Psi}\Psi_{,\nu}), \tag{48}$$

$$(*P^{\mu\nu}_{e})_{,\nu} = (\bar{\Phi}_{e}\gamma^{5}S^{\mu\nu}\Phi_{e})_{,\nu} = 0 \tag{49}$$

$$(P^{\mu\nu}_{m})_{,\nu} = (\bar{\Phi}_{m}S^{\mu\nu}\Phi_{m})_{,\nu} = 0 \tag{50}$$

$$(*P^{\mu\nu}_{m})_{,\nu} = (\bar{\Phi}_{m}\gamma^{5}S^{\mu\nu}\Phi_{m})_{,\nu} = \eta^{\mu\nu}I_{m}(\bar{\Psi}\gamma^{5}\Psi_{,\nu}) \tag{51}$$

$$(\Pi^{\mu\nu}_{e})_{,\nu} = (\bar{\chi}_{e}S^{\mu\nu}\chi_{e})_{,\nu} = I_{m}(\bar{\Psi}\gamma^{\mu}\Psi) \tag{52}$$

$$(*\Pi^{\mu\nu}_{e})_{,\nu} = (\bar{\chi}_{e}\gamma^{5}S^{\mu\nu}\chi_{e})_{,\nu} = 0 \tag{53}$$

$$(\Pi^{\mu\nu}_{m})_{,\nu} = (\bar{\chi}_{m}S^{\mu\nu}\chi_{m})_{,\nu} = 0 \tag{54}$$

$$(*\Pi^{\mu\nu}_{m})_{,\nu} = (\bar{\chi}_{m}\gamma^{5}S^{\mu\nu}\chi_{m})_{,\nu} = -in(\bar{\Psi}\gamma^{5}\gamma^{\mu}\Psi) \tag{55}$$

The subscripts e and m denote electric and monopolar, respectively, so that both the Mpt and mpt are made up of two parts: the macroscopic electronic polarization tensor (Mept) and the macroscopic monopolar polarization tensor (Mmpt), and the microscopic electronic polarization tensor (mept) and the microscopic monopolar polarization tensor (mmpt), defined by the pairs of equations (48)-(49), (50)-(51), (52)-(53), and (54)-(55).

The ordinary Dirac equation for the free electron is therefore associated with an electromagnetic field in which the Mpt has been neglected as well as the mmpt and in the absence of sources.

It is interesting to note that, still in the absence of free sources, by neglecting both the Mpt as well as the mpt, the generalized Dirac equation (20) reduces to

$$\gamma^{\mu}\Psi_{,\mu}=0 \tag{56}$$

which is the Dirac equation for the neutrino, so that the bare field appears to be the field associated with the neutrino.

The physical meaning of this result, as well as that of the generalized Dirac equation (20), is left to subsequent papers.

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